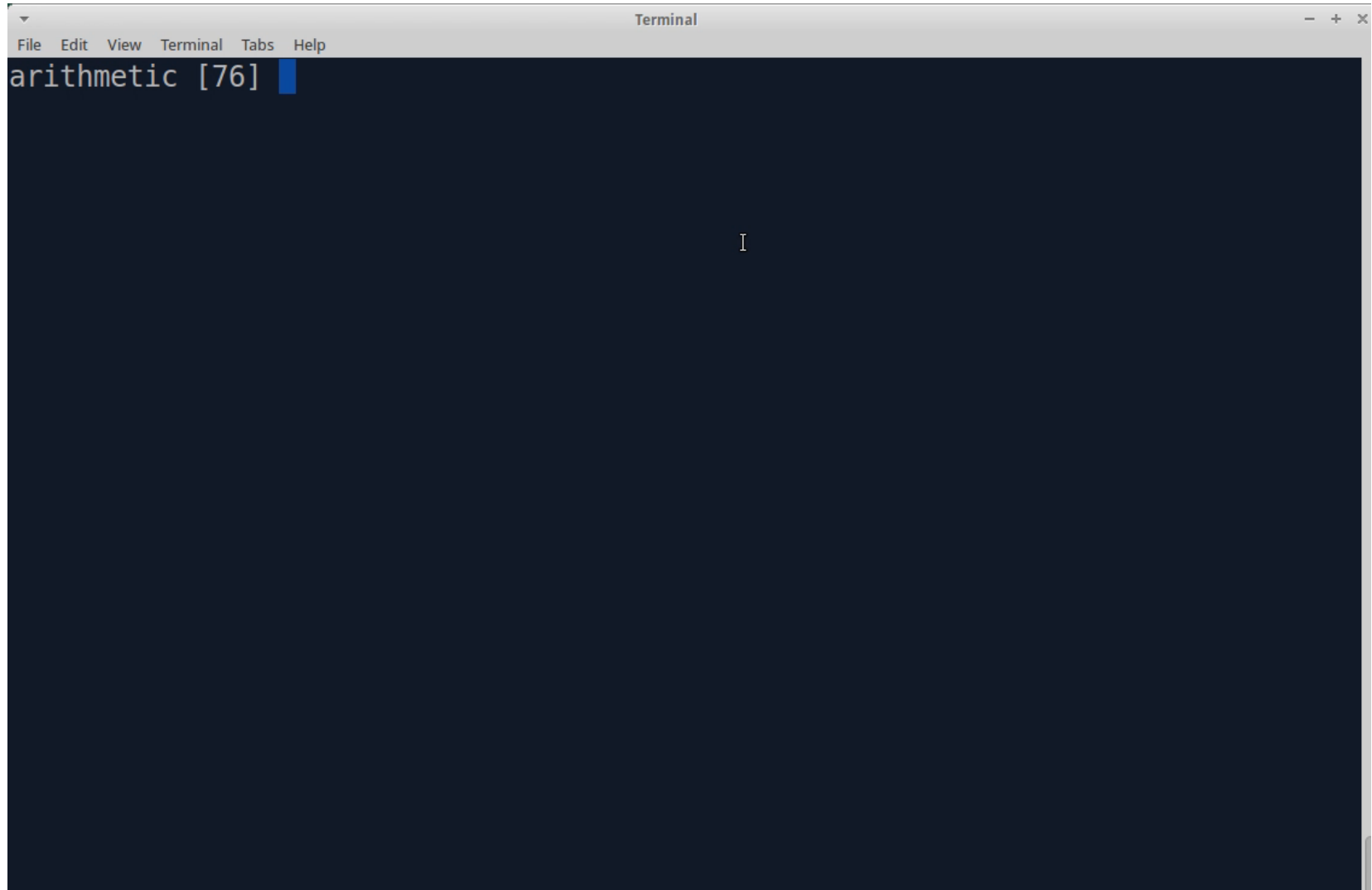




Arithmetic

- Learning Objectives
 - Explain how to encode both positive and negative numbers in binary
 - Add and subtract binary numbers
 - Explain why 2's complement is a good representation



```
Terminal
File Edit View Terminal Tabs Help
Command 'ls' from package 'coreutils' (main)
Command 'lsh' from package 'lsh-client' (universe)
ls]: command not found
arithmetic [81] pwd
/home/ubuntu/cs61/cs61-videos/arithmetic
arithmetic [82] ls
Makefile  hexdump.h  signs      signs.o  signs2.c  signs3    signs3.o  signs4.c
hexdump.c hexdump.o  signs.c    signs2   signs2.o  signs3.c  signs4    signs4.o
arithmetic [83] more signs2.c
#include <stdio.h>
#include "hexdump.h"

int
main(int argc, char ** argv)
{
    /* Suppress compiler warnings. */
    (void)argc;
    (void)argv;

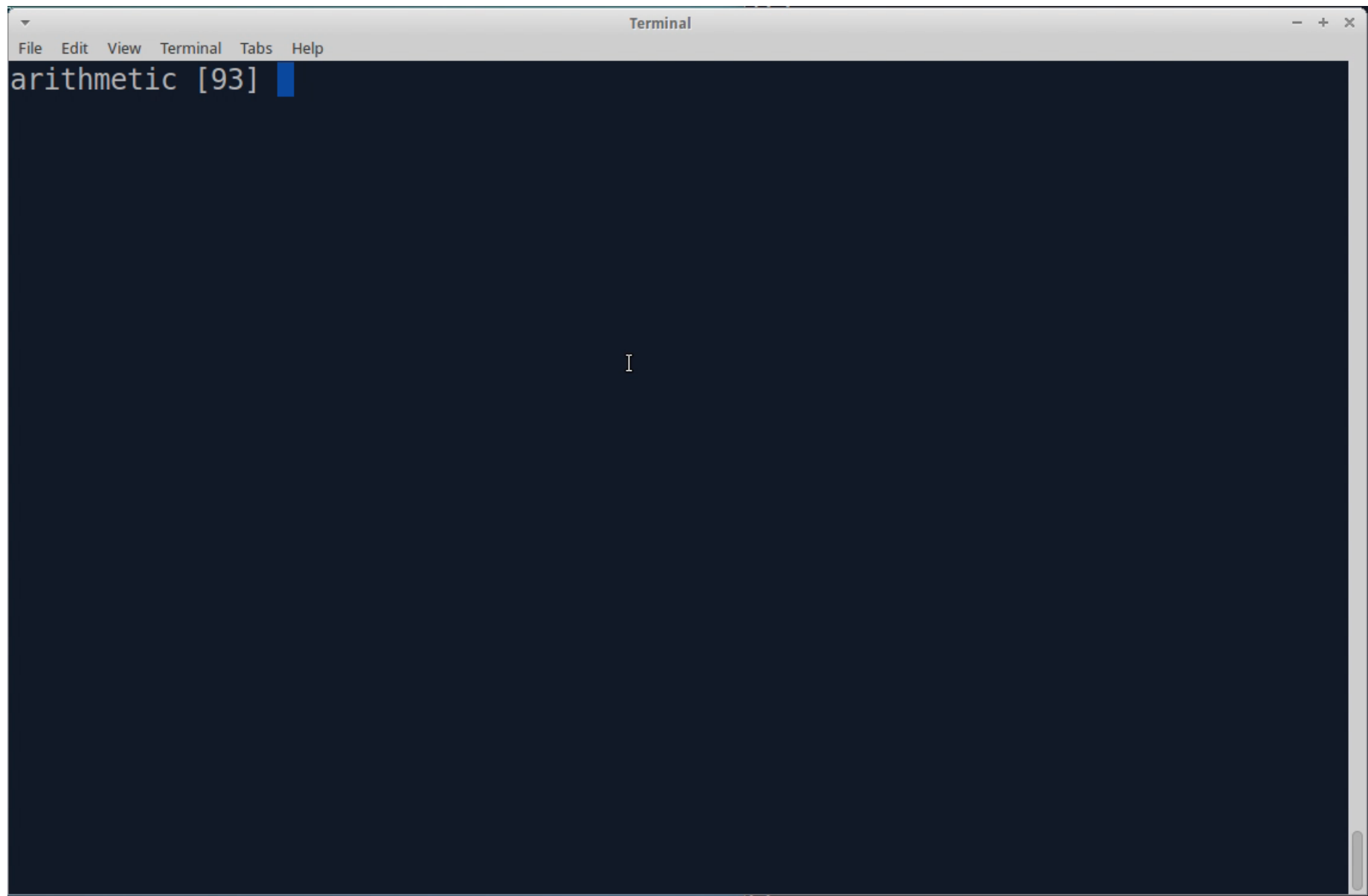
    int negs[4] = {-1, -2, -3, -4};

    hexdump(negs, sizeof(negs));
}
arithmetic [84]
```



Negative Numbers

- What do we know so far?
 - Negative (small) numbers appear very large (they start with 0xff).
 - Counting in negative numbers does seem to follow a regular counting pattern.
- You might hypothesize that negative numbers are positive numbers with the bits flipped.
- How might you test that?





2's Complement Arithmetic

- We can formalize what we observed, simply by examining values in memory:
- We represent $-n$ by taking the inverse of n ($\sim n$) and adding 1 to it:
 - $-n = (\sim n) + 1$
- Example (in 8 bits):

- $N = 5; 5 = \underline{00000101}$ $0x05$
- $\sim N = \underline{11111010}$ $(0xFA)$
- $\sim N + 1 = \underline{11111011} = 0xfb = -5$
 $\begin{array}{r} 00000101 \\ \hline 11111011 \\ \hline 11111100 \end{array}$



Why 2's Complement

- “It makes arithmetic operations just work.”
- How do we add binary numbers?

- $0 + 0 = 0$
- $0 + 1 = 1$
- $1 + 1 = 0$ (carry the 1)

- Example (again in 8 bits)

- $35 + 5$

00100011	0x23
00000101	0x05
<hr/>	<hr/>
00101000	0x28
<hr/>	
0x28	



Addition with Negative Numbers

- Let's see what happens when we try to add two negative numbers.

- Example: $-3 + -10$

$$\begin{array}{r}
 (-3) \overset{1}{1} \overset{1}{1} \overset{1}{1} \overset{1}{1} \overset{1}{1} \overset{1}{1} \overset{1}{1} \overset{1}{1} \overset{1}{1} \overset{1}{1} \\
 (-10) \overset{1}{1} \overset{1}{1} \overset{1}{1} \overset{1}{1} \overset{1}{1} \overset{1}{1} \overset{1}{1} \overset{1}{1} \overset{1}{1} \overset{1}{1} \\
 \hline
 11110011 \Rightarrow (-13)
 \end{array}$$

$0000\ 6011 + 0000\ 1010 = 0000\ 1101$
 $1111\ 1100 + 1111\ 0101 = 1111\ 0110$
 $1111\ 1101 + 1111\ 0110 = 1111\ 1101$ (with carry 13)

- Final check: $-1 + 1$ ought to equal 0!

$$\begin{array}{r}
 (-1) \overset{1}{1} \overset{1}{1} \overset{1}{1} \overset{1}{1} \overset{1}{1} \overset{1}{1} \overset{1}{1} \overset{1}{1} \overset{1}{1} \overset{1}{1} \\
 +1\ 00000001 \\
 \hline
 00000000
 \end{array}$$

